The M. Smithson was James Smithson, then a man of forty-four, the natural son of Hugh Smithson, later Duke of Northumberland. It may have been with his thoughts upon the bar sinister that he afterwards wrote:

"The best blood of England flows in my veins. On my father's side I am a Northumberland, on my mother's I am related to kings; but this avails me not. My name shall live in the memory of men when the titles of the Northumberlands and the Percys are extinct and forgotten."

Smithson spent most of his time on the Continent, and, evidently in the conquest of northern Germany, he had fallen into Napoleon's hands as a civilian prisoner of war. Delambre wrote this letter on the day that the emperor was hastening to the Battle of Ratisbon, where he defeated the Archduke Charles of Austria. This was seventeen years before Smithson made his will (October 23, 1826), and it is interesting to speculate upon the question of the founding of the Smithsonian Institution if Delambre had not written the letter.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

NEW QUESTION.

42. In connection with the questions of Kakeya [*1920*, 256], Professor W. B. Ford is led to the following inquiry: A line-segment AB is to be moved in its plane to a new position $A'B'$. How should this be done in order that the area generated may, to the greatest extent possible, be passed over three times?

Professor Ford has proved that if the generated area is to be passed over, to the greatest possible extent, but *two* times, AB should be rotated about the intersection of the perpendicular bisectors of AB and $A'B'$.

DISCUSSIONS.

Professor Campbell considers below the conditions under which the expression $P(x, y)dx + Q(x, y)dy$ represents an exact differential. The ordinary form of the criterion is $\partial Q/\partial x = \partial P/\partial y$, and involves assumptions about the derivatives of P and Q. Professor Campbell gives a form of necessary and sufficient condition which is applicable even though these derivatives fail to exist. The condition which he derives involves forms which are usually explicitly used in the proof of the ordinary theorem; but it does not seem that his statement of the condition as an end in itself, is found in the literature. He gives also a generalization to the case of n variables. It seems that the restriction to a rectangular region, alluded to in a footnote, is essential for the accuracy of the proof.

Professor McKelvey contributes some remarks on a universally troublesome question,—the presentation of the theory of limits in secondary schools. While his indication that it is never of importance whether or not a variable reaches its limit may require occasional modification, such modification surely bears, not on the question of the general meaning of limit, but on the special problem

in hand. A more precise presentation of the idea of limit than is customary would greatly facilitate the use of the notion in college teaching.

Recent numbers of the MONTHLY have contained articles by Mr. Cheney [*1920*, 53] and Professor Lovitt [*1920*, 465] on geometric proofs of the law of tangents. A proof distinct from those as yet proposed is given in the last discussion this month by Professor Epperson.

I. ON EXACT DIFFERENTIALS.

By J. W. CAMPBELL, University of Alberta.

The criterion usually given that the differential $Pdx + Qdy$ shall be exact is

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$

The functions P and Q are therefore assumed to be differentiable, and they are also assumed to have the other properties necessary for the application of Green's theorem. The purpose of this note is to suggest, in this case and in the case of n variables, an integral condition in which P and Q do not necessarily satisfy the hypotheses of Green's theorem.

THEOREM I.[1] *The necessary and sufficient conditions that*

$$Pdx + Qdy \tag{1}$$

shall be an exact differential are that P and Q shall be integrable with regard to x and y, respectively, and that

$$\int_{x_0}^{x} P(x, y)dx + \int_{y_0}^{y} Q(x_0, y)dy \equiv \int_{y_0}^{y} Q(x, y)dy + \int_{x_0}^{x} P(x, y_0)dx, \tag{2}$$

where (x_0, y_0) is an arbitrary fixed point in the vicinity of which P and Q are integrable.

For if (1) is exact it must be of the form

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy,$$

whence

$$\frac{\partial u}{\partial x} = P, \qquad \frac{\partial u}{\partial y} = Q.$$

Therefore

$$\begin{aligned} u &= \int_{x_0}^{x} P(x, y)dx + f(y) \\ &= \int_{y_0}^{y} Q(x, y)dy + g(x), \end{aligned} \tag{3}$$

where f and g are arbitrary functions of integration.

[1] It is assumed that the region under consideration is rectangular.—EDITOR.